**Summary of the Monte Carlo method for gas storage**

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**Monte Carlo Simulation Method**

In a Monte Carlo simulation, one attempts to generate prices according to a distribution. We have the forward price and volatility curves. This translates into a distribution for a price vector that has a mean (forward price curve), a standard deviation (forward volatility curve), and a correlation matrix. Using a random number generator, one can generate a large number of sample vectors according to the distribution. For each of these price vectors, we have to value the storage deal. Taking the average over all these sample values provides us with the value of storage.

A large number of simulations are required to reduce sample error. As the number of simulation runs increases, the size of the sample error decreases by the square root of the number of runs. As a result, it takes a long time to reduce the size of the error. Techniques used in Monte Carlo simulation (*e.g.*, Control Variate method) can be used to reduce the sample error, but it is hard to apply the control variate method to the case of natural-gas storage. Furthermore, calculating the moments requires special care. When one uses the Monte Carlo simulation to calculate the price deltas (or first moments), one needs to take special care.

To calculate the delta, the value of the deal usually is calculated, then the price is “tweaked” by a small amount and the new value is calculated. The difference yields the price delta. The problem that arises is that each of the calculations has a sample error. How much of the price difference is due to the change in prices and how much is due to the sample error?

Monte Carlo methods are inappropriate for valuing American options. A fundamental problem arises with the use of Monte Carlo simulation for valuing American options. The problem arises from the way the prices are generated in a Monte Carlo simulation. In a Monte Carlo simulation, the prices are generated ex post—in other words, the model assumes that the deal has been completed, it then looks at the settlement prices for each of the time periods, and it uses this price vector to determine the optimal injection and withdrawal schedule. In an American option—where an option can be exercised early for each given possible sample price path—the holder cannot “peek” along the path into the future to determine what the settlement price will be in order to determine whether or not to exercise. The user has to look at the information available only at the time of exercise. Hence, the value obtained using the Monte Carlo method will always be higher because of the advantage that this method has over the reality. The modeler now has to adjust the parameters so that the values are consistent with those being seen in practice. It should also be pointed out that the Monte Carlo method can produce an inaccurate representation of how the value of storage is monetized.

In the Monte Carlo method, the assumption is that the information is available to the model after the fact. So the algorithm goes through the following steps:

* Generate a random price path;
* Determine the optimal injection withdrawal schedule for each path;
* Perform this step a number of times; and
* Take the average over all such paths to obtain the value.

The algorithm looks at a price path, determines the optimal schedule, and obtains the value. This is not the case in practice. The problem here is that in reality, all the decisions are not made simultaneously. On this sample path, one cannot implement the rolling intrinsic hedge strategy. If you were required to be delta neutral, this would require that any gas injected into the ground has a corresponding sale out in the future, but one could change the timing of the sale anytime before the gas was actually withdrawn from the ground. In conclusion, the Monte Carlo simulation fails to capture the true dynamics of storage valuation.

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Different optimization algorithms for maximizing the profits from natural gas storage usage have been proposed in the literature. Generally two approaches can be distinguished. While solving a Bellman equation provides a closed form solution given certain price generating functions, Monte Carlo simulations are very flexible with respect to constraints and price models but have no analytic solution. To cope with a nonstandard price function (reversion to moving mean) as well as nonlinearities in constraints and cost we follow Boogert and de Jong (2006) applying a Least Square Monte Carlo approach to natural gas storage contracts. Since identifying the optimal storage strategy is comparable to locating the exercise date of American options, Boogert and de Jong (2006) apply an option valuation algorithm proposed by Longstaff and Schwartz (2001). The general idea of the concept is to optimize storage usage decisions backwards in time using a discrete (daily) time grid, a discrete volume grid and n simulated price paths. The volume grid stretches from minimum to maximum storage level at equal distance volume steps: Volmin:VolStep:Volmax. These volume steps are defined to approximately represent a tenth of the daily decision spectrum (i.e., the difference of maximum injections and maximum withdrawals). Thus, at each day and volume combination, around ten different decisions are possible (cf. Figure 1.4.2). Time-values for a discrete set of allowed strategies are compared at each decision making point. Consequently, we first define a termination date and the payoff function at this date. We set the termination date T=t0+365 (that is one year after the start date), and the payoff function at T is defined depending on the volume in storage at termination date (VolT ). If the volume exceeds a desired level (Vol∗ ) the payoff is zero. We assume that a user has to pay a punishment of double the time-value of the missing volume if the critical level Vol∗ is undercut.

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